Numerical simulations of rotating Rayleigh-Bénard convection

Richard J.A.M. Stevens, Herman J.H. Clercx, and Detlef Lohse

Abstract Here we present results from direct numerical simulation (DNS) in which we solved the three-dimensional Navier-Stokes equations within the Boussinesq approximation in a cylindrical domain. Due to rotation, rising or falling plumes of hot or cold fluid are stretched into vertical vortices that suck fluid out of the thermal boundary layers (BLs) adjacent to the bottom and top plates. This process, called Ekman pumping, contributes to the vertical heat flux. We show that this process is strongly influenced by the *Pr* number. When the rotation rate is increased (*Ro* decreased) the heat transfer first increases due to Ekman pumping, before it rapidly falls down at even stronger rotation. When there is no or very weak rotation the large scale circulation (LSC) is dominant. For stronger rotation ($Ro < 2$) Ekman pumping becomes dominant. It turns out that there is an optimal *Pr* number (with fixed *Ra* and *Ro*) for the heat transport enhancement. This maximum is found because the efficiency of Ekman pumping is limited for high and low *Pr*. For low *Pr* the efficiency is limited by the large thermal diffusivity, i.e. the heat that is sucked out of the thermal BL spreads out horizontally. For high *Pr* the efficiency is limited because the temperature of the fluid that is sucked out of the thermal BLs is relatively low.

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The Rayleigh-Bénard (RB) system is relevant to astro- and geophysical phenomena, including convection in the ocean, the Earth's outer core, and the outer layer of the Sun. The dimensionless heat transfer (the Nusselt number Nu) in the system depends on the Rayleigh number $Ra = \beta g \Delta L^3/(\nu\kappa)$ and the Prandtl number $Pr = \nu/\kappa$. Here, β is the thermal expansion coefficient, g the gravitational acceleration, Δ the temperature difference between the bottom and top, and ν and κ the kinematic viscosity and the thermal diffusivity, respectively. The rotation rate H is used in the form of the Rossby number $Ro = (\beta g \Delta/L)/(2H)$. The key question is: How does the heat transfer depend on rotation and the other two control parameters: $Nu(Ra,Pr,Ro)$? Here we will answer this question by giving a summary of our results presented in [1, 2, 3].

We present both experimental measurements [1] (fig. 1) and results from direct numerical simulation (DNS) (fig. 4). They cover different but overlapping parameter ranges and thus complement each other. The convection apparatus that was used in the experiments is described in detail as the "medium sample" in Ref. [4]. All measurements were made at constant imposed Δ and Ω, and fluid properties were evaluated at $T_m = (T_t + T_b)/2$ [1].

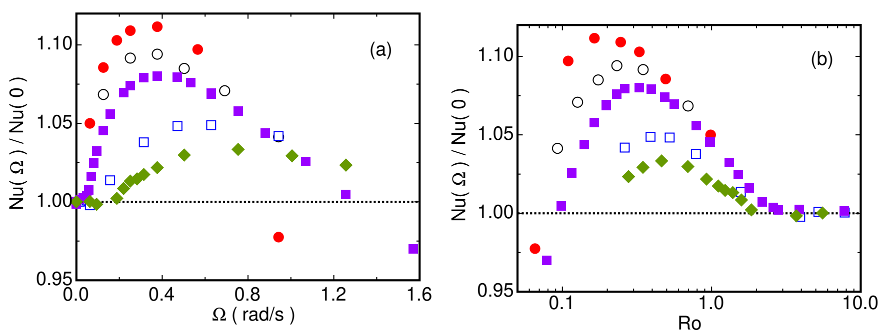

Fig. 1 The ratio of the Nusselt number $Nu(\Omega)$ in the presence of rotation to $Nu(\Omega = 0)$ for $Pr = 4.38$ ($T_m = 40.00°$C). (a): Results as a function of the rotation rate in rad/sec. (b): The same results as a function of the Rossby number Ro on a logarithmic scale. Solid circles: $Ra = 5.6 \times 10^8$ ($\Delta = 1.00$ K). Open circles: $Ra = 1.2 \times 10^9$ ($\Delta = 2.00$ K). Solid squares: $Ra = 2.2 \times 10^9$ ($\Delta = 4.00$ K). Open squares: $Ra = 8.9 \times 10^9$ ($\Delta = 16.00$ K). Solid diamonds: $Ra = 1.8 \times 10^{10}$ ($\Delta = 32.00$ K). Here and in fig. 4 experimental uncertainties are typically no larger than the size of the symbols. Figure adapted from [1].

In the DNS we solved the three-dimensional Navier-Stokes equations within the Boussinesq approximation in a three dimensional cylindrical domain

$$\frac{D\mathbf{u}}{Dt} = -\nabla P + \left(\frac{Pr}{Ra}\right)^{1/2} \nabla^2 \mathbf{u} + \theta \hat{z} - \frac{1}{Ro} \hat{z} \times \mathbf{u}, \tag{1}$$

$$\frac{D\theta}{Dt} = \frac{1}{(PrRa)^{1/2}}\nabla^2\theta, \tag{2}$$

with $\nabla \cdot \mathbf{u} = 0$. Here $\widehat{z}$ is the unit vector pointing in the opposite direction to gravity, $\mathbf{u}$ the velocity vector, and θ the non-dimensional temperature, $0 \leq \theta \leq 1$. Finally, P is the reduced pressure (separated from its hydrostatic contribution, but containing the centripetal contributions): $P = p - r^2/(8Ro^2)$, with r the distance to the rotation axis. The equations have been made non-dimensional by using, next to L and Δ, the free-fall velocity $U = \sqrt{\beta g \Delta L}$. The resolution is sufficient to represent the small scales both inside the bulk of turbulence and in the boundary layers (BLs) (where the grid-point density has been enhanced) for the parameters employed here. Nu is calculated as in ref. [5] and its statistical convergence has been verified [1, 2].

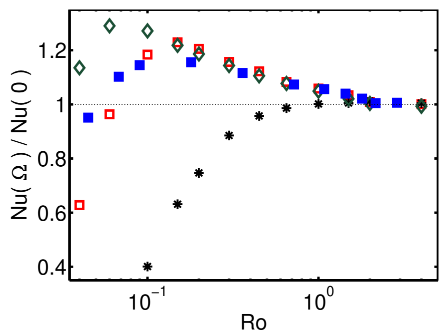

Fig. 2 The ratio $Nu(\Omega)/Nu(\Omega = 0)$ as function of Ro on a logarithmic scale. Blue solid squares: $Ra = 1 \times 10^9$ and $Pr = 6.4$ (DNS) [6]. Open squares: $Ra = 1 \times 10^8$ and $Pr = 6.4$ (DNS). Open diamonds: $Ra = 1 \times 10^8$ and $Pr = 20$ (DNS). Stars: $Ra = 1 \times 10^8$ and $Pr = 0.7$ (DNS). Figure adapted from [1]. Additional experimental and DNS data for $Ra = 2.73 \times 10^8$ and $Pr = 6.26$ can be found in figure 3 of ref. [1].

In [1] we showed that the heat-flux enhancement can be as large as 30% and depends strongly on Pr and Ra (fig. 1 and 4). The increased heat transfer is due to Ekman pumping; i.e. due to the rotation, rising or falling plumes of hot or cold fluid are stretched into vertical vortices that suck fluid out of the thermal BLs adjacent to the bottom and top plates (fig. 3b). For $Pr = 6.4$ thermal structures are basically confined to these vortices, whereas for $Pr = 0.7$ they are much shorter and broadened, because the larger thermal diffusion, which makes the Ekman pumping ineffective. This results in a decreasing heat transfer at lower Pr (fig. 3a). Along the same line of reasoning one can explain the reduced effect of Ekman pumping at higher Ra, see figure 1, because with increasing Ra the turbulence is enhanced which leads to a larger eddy thermal diffusivity, promoting a homogeneous mean temperature in the bulk.

In [3] we found that there is an optimal Pr number for the heat transfer enhancement at a fixed Ro, see figure 4. The observation that there is a Pr number for which the heat transfer enhancement is largest suggests that there should be at least two

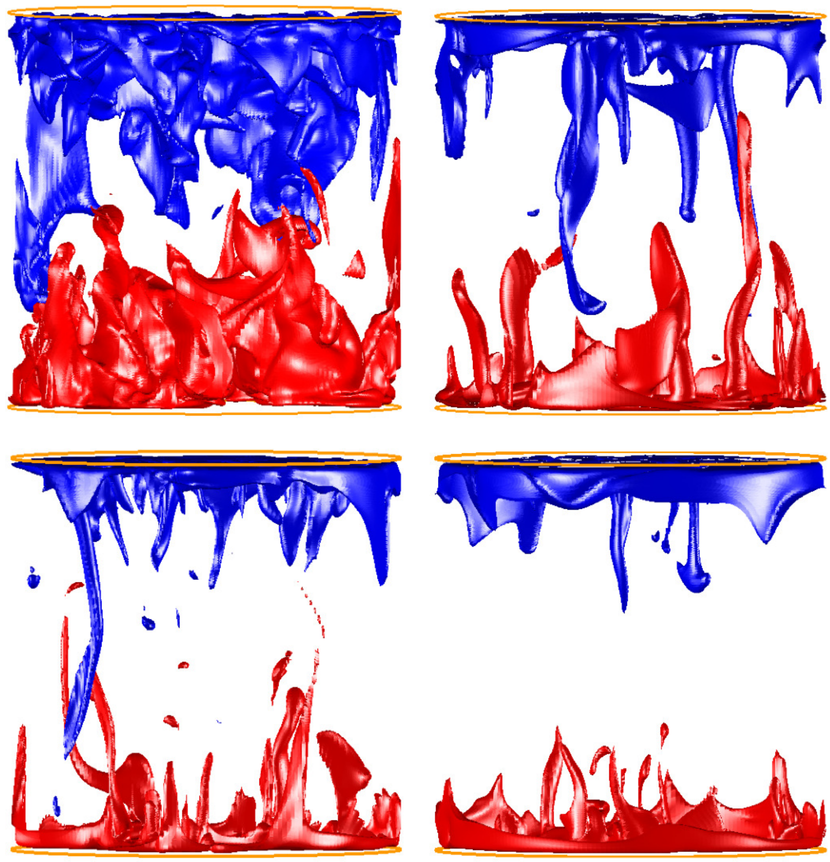

Fig. 3 3D visualization of the temperature isosurfaces in the cylindrical sample of $\Gamma = 1$ at 0.65Δ (originating from bottom) and 0.35Δ (originating from top), respectively, for $Pr = 0.7$ (left upper plot) and $Pr = 6.4$ (right upper plot), $Pr = 20$ (left lower plot), and $Pr = 55$ (right lower plot) for $Ra = 10^8$ and $Ro = 0.30$. The snapshots were taken in the respective statistically stationary regimes. Figure adapted from [3].

competing effects, which strongly depend on the Pr number, that control the effect of Ekman pumping. One wonders why Ekman pumping becomes less efficient for higher Pr numbers. Clearly, it must have a different origin than the reduced effect of Ekman pumping at lower Pr. Indeed, there is an important difference between the high and the low Pr number regime, namely the relation between the thickness of the thermal and the kinetic boundary layers (BL). For the low Pr number regime the kinetic BL is thinner than the thermal BL and therefore the fluid that is sucked into the vertical vortices is very hot. When the Pr number is too low this heat will spread out in the middle of the cell due to the large thermal diffusivity. For somewhat higher Pr number the fluid that is sucked out of the thermal BL is still sufficiently hot and due to the smaller thermal diffusivity the heat can travel very far from the plate in

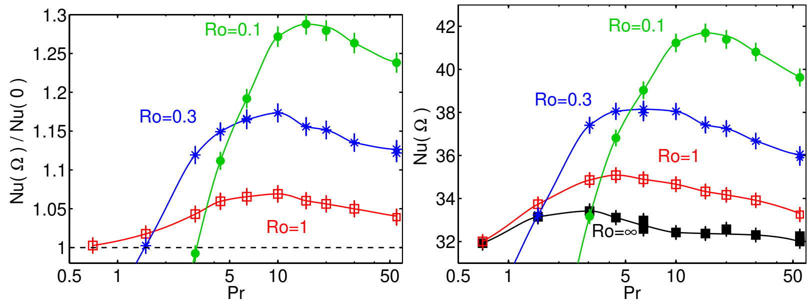

Fig. 4 The heat transfer as function of Pr on a logarithmic scale for $Ra = 1 \times 10^8$. Solid squares, open squares, stars, and solid circles indicate the results for $Ro = \infty$, $Ro = 1.0$, $Ro = 0.3$, and $Ro = 0.1$, respectively. Figure adapted from [3].

the vertical vortices. In this way Ekman pumping can increase the heat transfer for moderate Pr. In the high Pr number regime the kinetic BL is much thicker than the thermal BL. Therefore, the Ekman vortices forming in the bulk do not reach the thermal BL and hence the temperature of the fluid that enters the vertical vortices is much lower. This is schematically shown in figure 5.

An investigation of the temperature isosurfaces [1, 3] revealed long vertical vortices as suggested by Ekman-pumping at $Pr = 6.4$, while these structures are much shorter and broadened for the $Pr = 0.7$ case due to the larger thermal diffusivity at lower Pr [1]. In figure 3 we show the temperature isosurfaces at $Ro = 0.30$ for several Pr to identify the difference between the high and moderate Pr number cases. The figure reveals that the vertical transport of hot (cold) fluid away from the bottom (top) plate through the vertical vortex tubes is strongly reduced in the high-Pr number regime compared to the case with $Pr = 6.4$. This is illustrated in figure 3 where the threshold for the temperature isosurfaces is taken constant for all cases. Indeed, a closer investigation shows that the vortical structures for the higher Pr are approximately as long as for the $Pr = 6.4$ case. This confirms the view that the temperature of the fluid that is sucked into the Ekman vortices decreases with increasing Pr.

To summarize, we studied the Ra and Pr number dependence of the heat transport enhancement in rotating RB convection [1, 2, 3]. We showed that at a fixed Ro number there is a Pr number for which the heat transfer enhancement reaches a maximum. This is because Ekman pumping, which is responsible for the heat transfer enhancement, becomes less efficient when the Pr number becomes too small or too large. At small Pr numbers the effect of Ekman pumping is limited due to the large thermal diffusivity due to which the heat that is sucked out of the BLs rapidly spreads out in the bulk. At high Pr numbers the temperature of the fluid that is sucked into the vertical vortices near the bottom plate is much lower, because the thermal BL is much thinner than the kinetic BL, and therefore the effect of Ekman pumping is lower. Furthermore, heat transport enhancement also decreases with increasing Ra, because of a larger eddy thermal diffusivity.

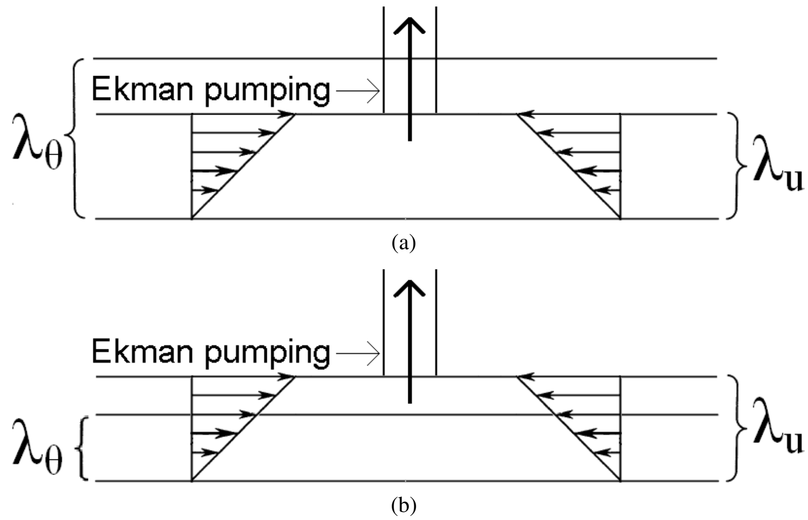

Fig. 5 a) Sketch for the low *Pr* number regime where the Ekman vortices reach the thermal BL. b) Sketch for the high *Pr* number regime where the Ekman vortices do not reach the thermal BL. Figure adapted from [3].

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